# Chapter 2: Financial Math

### Exercises 2.4

1. You can afford a $700 per month mortgage payment. You’ve found a 30-year loan at 5.5% interest.
2. How big of a loan can you afford?
3. How much total money will you pay the loan company?
4. How much of that money is interest?
5. Marie can afford a $250 per month car payment. She’s found a 5-year loan at 7% interest.
   1. How expensive of a car can she afford?
   2. How much total money will she pay the loan company?
   3. How much of that money is interest?
6. You want to buy a $25,000 car. The company is offering a 2% interest rate for 48 months (4 years). What will your monthly payments be?
7. You decide to finance a $12,000 car at 3% compounded monthly for 4 years. What will your monthly payments be? How much interest will you pay over the life of the loan?
8. You want to buy a $200,000 home. You plan to pay 10% as a down payment and take out a 30-year loan for the rest.
   1. How much is the loan amount going to be?
   2. What will your monthly payments be if the interest rate is 5%?
   3. What will your monthly payments be if the interest rate is 6%?
9. Lynn bought a $300,000 house, paying 10% down, and financing the rest at 6.5% interest for 30 years.
   1. Find her monthly payments.
   2. How much interest will she pay over the life of the loan?
   3. What percentage of your total payment was interest?
10. Emile bought a car for $24,000 three years ago. The loan had a 5-year term at 3% interest rate. How much does he still owe on the car?
11. A friend bought a house 15 years ago, taking out a $120,000 mortgage at 6% for 30 years. How much does she still owe on the mortgage?
12. Imagine you have $20,000 saved as a down payment on a house. You wish to take out a fixed-rate 30-year mortgage loan at 4% APR (remember that mortgage rates usually assume monthly compounding). If the maximum mortgage payment you can afford is $950 per month, then what is the maximum house price that you can afford? ADD
13. Suppose another mortgage lender offers you a fixed-rate 15-year mortgage at 2.95% APR. You have $20,000 saved as a down payment, and you can afford a maximum mortgage payment of $950 per month. You are interested in a certain house for sale, with firm selling price of $200,000. ADD
14. Find the monthly payment for this house. Can you afford it, under the terms of this lender?
15. (Challenge): Suppose this same lender offers to increase the APR by only 0.05%, for each additional year added to the loan period beyond 15 years (so that a 16-year loan would have 3.00% APR, and a 17-year loan would have 3.05% APR, and so on), up to a maximum loan period of 25 years. Given these terms, does any combination of APR and loan period exist that would let you afford the house? If so, state the *minimum* number of additional years needed, the total resulting loan period, the resulting APR, and the resulting monthly payment.
16. An annuity firm pays 5% APR compounded yearly, and offers an investor the following: Deposit $100,000 with them today, and then starting one year from today, you will receive ten equal annual payments, with zero balance remaining afterward in the account. If an investor accepts their offer, then find the following:
17. What will be the payment amount for each of the ten equal payments?
18. After ten years, how much interest will the investor have received, and what percentage of the total payment sum will represent interest? ADD
19. For twelve full years, and into an account that pays 3.5% compounded quarterly: Yanhong will either pay $1500 at the end of each calendar quarter, or depositing a single lump sum that will give the same future value amount.
20. If Yanhong chooses the single lump sum option, then how much will Yanhong need to deposit?
21. If Yanhong needs to have earned $100,000 in this account at the end of the twelve years, then the quarterly deposit amount will need to be increased. What would the new quarterly deposit amount need to be?
22. (Challenge): If Yanhong will make quarterly deposits into this account for the twelve years, but also has $8,000 to additionally deposit into this account right away: What would the new quarterly deposit amount need to be, so that the total balance after twelve years is $100,000? ADD
23. Assume you take out a 30-year mortgage loan for $250,000 at a fixed 4.5% APR.
24. What will be the amount of your monthly payment?
25. After the first ten years of payments, how much will remain on your loan balance?
26. After the first twenty years of payments, how much will remain on your loan balance?
27. Notice that the amount of the loan balance reduction during the second ten years, was very considerably bigger than the amount of the loan balance reduction during the first ten years. Why does the loan balance decrease at a faster and faster pace, the longer that the loan has been in repayment? ADD

**Answers for Exercises 2.4**

(Note: Spreadsheet input is shown in a cell box if viewing in Microsoft Word)

**1. a)** , which gives *P* ≈ $123,285.23

or =PV(5.5%/12,12\*30,-700) [Note 700 is entered as negative, to signify a *payment*]

**b)** 700 \* 12 \* 30 dollars, or $252,000.00 in total payments to the loan company.

**c)** Interest will be the difference between the total payments, and the amount borrowed.

So the interest on this loan is $252,000.00 - $123,285.23 = $128,714.77.

**2. a)** , which gives *P* ≈ $12,625.50

or =PV(7%/12,12\*5,-250)

**b)** 250 \* 12 \* 5 dollars, or $15,000.00 in total payments to the loan company.

**c)** Interest will be the difference between the total payments, and the amount borrowed.

So the interest on this loan is $15,000.00 - $12,625.50 = $2,374.50.

**3.** , which gives *d* ≈ $542.38

or =PMT(2%/12,48,25000)

**4.** , which gives *d* ≈ $265.61

or =PMT(3%/12,48,12000)

The interest will be the total amount paid, minus the amount of the loan.

So the interest here is ($265.61 \* 48) – $12,000 = $749.28

**5. a)** The loan amount will be 90% of $200,000.00

= (0.9 \* $200,000.00)

= $180,000.00

**b)** , which gives *d* ≈ $966.28

or =PMT(5%/12,12\*30,180000)

**c)** , which gives *d* ≈ $1,079.19

or =PMT(6%/12,12\*30,180000)

**6. a)** , which gives *d* ≈ $1,706.58

or =PMT(6.5%/12,12\*30,270000)

**b)** The total of all loan payments will be (12 \* 30 \* $1,706.58) = $614,368.80.

So the total interest paid will be $614,368.80 – $270,000.00 = $344,368.80.

**c)** ($344,368.80 / $614,368.80), or ≈ 56.0525%

**7.** First, we need to find out the amount of the monthly payments for this loan.



or =PMT(3%/12,12\*5,24000) which gives *d* ≈ $431.25

The amount still owed three years later, is the present value of the two years of remaining payments on the loan.

, which gives *P* ≈ $10,033.45

or =PV(3%/12,12\*2,-431.25)

**8.** First, we need to find out the amount of the monthly payments for this loan.



or =PMT(6%/12,12\*30,120000) which gives *d* ≈ $719.46

The amount still owed fifteen years later, is the present value of the fifteen years of remaining payments on the loan.

, which gives *P* ≈ $85,258.54

or =PV(6%/12,12\*15,-719.46)

***Use a spreadsheet (not formulas) to answer problems 9 through 13 below.***

**9.** =PV(4%/12,12\*30,950) which gives $198,988.18.

Now add $20,000 to this balance, which gives $218,988.18.

**10. a)** =PMT(2.95%/12,12\*15,180000,0) which gives $1,238.72.

You cannot afford this home.

**b)** Using trial-and-error: The minimum number of additional years is 8.

There is a 23 year loan period, 3.35% APR, and $936.24 monthly payment.

**11. a)** =PMT(5%,10,100000,0) which gives $12,950.46.

**b)** The total payments will be $129,504.60.

The interest is the amount over the $100,000 initial investment, or $29,504.60.

The percentage of the total payment sum representing interest will be 100\*(29,504.60/129,504.60)%, or approximately 22.7827%.

**12. a)** =PV(3.5%/4,4\*12,1500) which gives $58,586.02.

**b)** =PMT(3.5%/4,4\*12,0,100000) which gives $1,685.34.

**c)** =PMT(3.5%/4,4\*12,-8000,100000) which gives $1,480.51.

(Note 8000 is input as negative, to signify a payment.)

Note this answer can also be found by subtracting the future value of $8000 here, which equals $12,153.47, from the required ending balance of $100,000 (which leaves $87,846.47) and then using the PMT function:

=PMT(3.5%/4,4\*12,0,100000-12153.47) which again gives $1,480.51.

This can also be done in nested fashion (be careful to add the negative FV quantity): =PMT(3.5%/4,4\*12,0,100000+FV(3.5%/4,4\*12,0,8000)).

**13. a)** =PMT(4.5%/12,12\*30,250000) which gives $1,266.71.

**b)** This will be the present value of the remaining 240 loan payments:

=PV(4.5%/12,12\*20,1266.71) which gives $200,223.07.

**c)** This will be the present value of the remaining 120 loan payments:

=PV(4.5%/12,12\*10,1266.71) which gives $122,223.99.

**d)** At the beginning of the repayment period, most of each payment goes to interest (thus the loan balance reduces very slowly at first). Over time, more of each payment shifts to principal, and less to interest. At the end of the loan repayment period, nearly all the payment is going to principal.